A FINITE DIFFERENCE METHOD FOR THE EXCITATION OF A DIGITAL WAVEGUIDE STRING MODEL

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Who's Who

3MediaLabs: Multimedia Information Processing

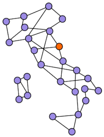
Made of two research sub-groups: *A3LAB* and *SEMEDIA*.
LEADER: Prof. Francesco Piazza

A3LAB: DSP Algorithms and Adaptive systems for Audio applications

- *Data* Processing Approach
- People: 2 Assistant Professors, 4 PostDocs, 4 Phd Students

SEMEDIA: Semantic Web and Multimedia

- *MetaData* Processing Approach
- People: 3 PostDocs



Wave Equation Solutions

Ideal String Wave Equation

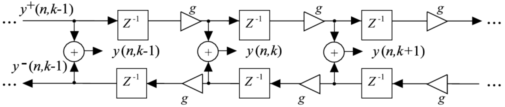
$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \tag{1}$$

(d'Alembert, 1747)

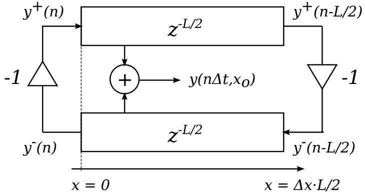
1D DWG

General solution found by d'Alembert in 1747:

$$y(k, n+1) = gy^{+}(k-1, n) + gy^{-}(k+1, n) \qquad (2)$$



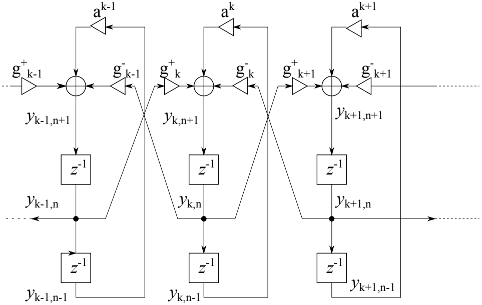
Consolidating the delays and gains yields extremely computational efficient solutions



First Order FDTD Scheme (FOFS)

Central differences scheme:

$$y_{k,n+1} = g_k^+ y_{k-1,n} + g_k^- y_{k+1,n} + a_k y_{k,n-1} \quad (3)$$



Losses can be frequency-dependent [Karjalainen, 2002]

Stiff strings are also characterized by dispersion, the new wave equation is:

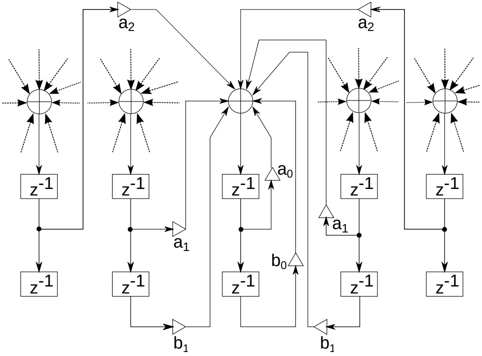
$$\frac{\partial^2 y}{\partial t^2} = c\frac{\partial^2 y}{\partial x^2} - \lambda\frac{\partial^4 y}{\partial x^4} - 2\sigma_0\frac{\partial y}{\partial t} + 2\sigma_1\frac{\partial y}{\partial t}\frac{\partial^2 y}{\partial x^2} \quad (4)$$

- λ stiffness,
- σ_0 f-independent loss
- σ_1 f-dependent loss

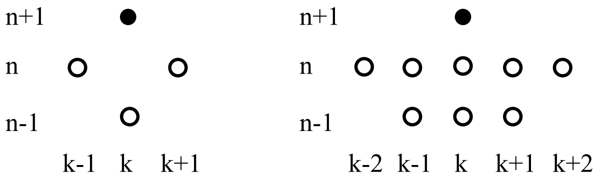
Second order FDTD scheme (SOFS)

A numerical scheme for the PDE (4) after [Bilbao, 2010]

$$\begin{aligned} y_{k,n+1} = a_0 y_{k,n} + a_1(y_{k+1,n} + y_{k-1,n}) + \\ + a_2(y_{k+2,n} + y_{k-2,n}) + b_0 y_{k,n-1} + \\ + b_1(y_{k+1,n-1} + y_{k-1,n-1}) \end{aligned} \quad (5)$$



FOFS vs. SOFS



FOFS for (1) SOFS for (4)

Pros and Cons

DWG

- Efficiency
- Numerical stability
- Flexible addition of DSP blocks

- Departure from underlying physics
- Characterization based on audio analysis

FDTD

- Characterization based on physical measurement
- Minimal precomputation of parameters
- Emulation of distributed nonlinear contact

- Computational cost (not an issue anymore)
- Coefficients design

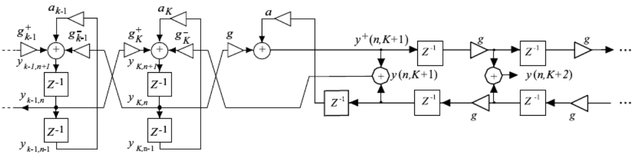
Interfacing DWG and FDTD

Previous works

Mixed modeling

- M. Karjalainen, C. Erkut, and L. Savioja, "Compilation of unified physical models for efficient sound synthesis", in Acoustics, Speech, and Signal Processing, 2003. Proceedings.(ICASSP 03) 2003 IEEE International Conference on. IEEE, 2003, vol. 5, pp. V-433.
- M. Karjalainen and C. Erkut, "Digital Waveguides versus Finite Difference Structures: Equivalence and Mixed Modeling", EURASIP Journal on Advances in Signal Processing, no. 7, pp. 978-989, 2004.

1-D First Order case



(adapted from [Erkut, 2002])

Mixed Modeling Goals

- Unified approach (1 POV)
- Modular modeling (many
- FDTD features at a lower cost

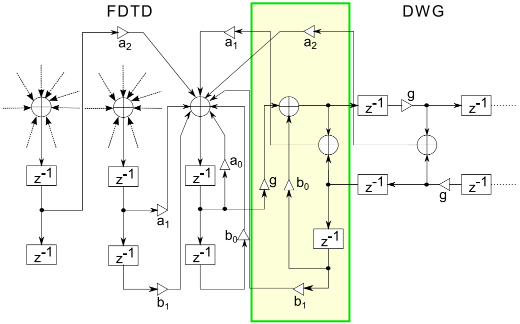
SOFS-DWG Interface Conditions for Matching

$$
\begin{aligned}
y_{k,n+1} = a_0 y_{k,n} + \\
+a_1(y_{k-1,n} + y_{k+1,n}) + \\
+b_1(y_{k-1,n-1} + y_{k+1,n-1}) + \\
+b_0 y_{k,n-1} + a_2(y_{k-2,n} + y_{k+2,n})
\end{aligned} \quad (6)
$$

with k: last FDTD spatial point

$$y^{+}_{k+1,n} = y^{-}_{k+1,n-1} + g y_{k,n} \quad (7)$$

with k+1: first DWG spatial point



Case Study

Hohner Clavinet D6

Mechanical Action

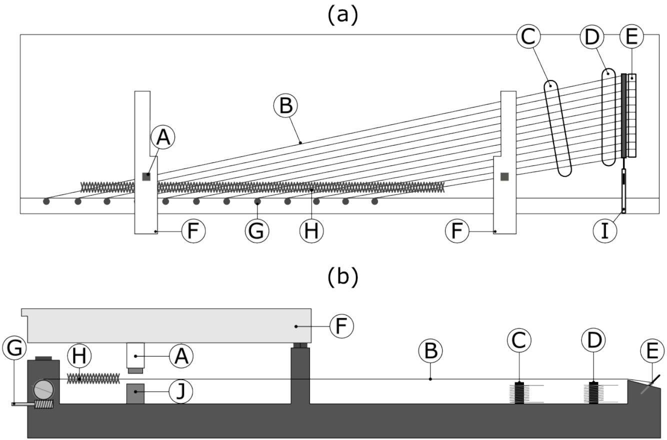
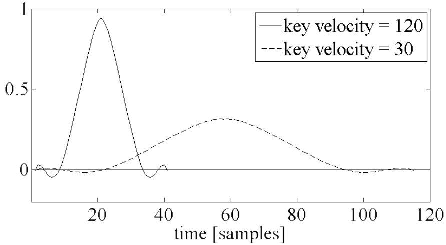


Figure : (a) Top view - (b) Side view

Polynomial Pulse Model (PPM)

$$f(x) = a_p x^p + a_{p-1} x^{p-1} + ... + a_1 x + a_0 \quad (8)$$



Excitation pulse signal for *piano* and *forte* tones

Spectral Envelope Model (SEM)

$$E = \alpha_1 E_1 + (1 - \alpha) E_2 \quad (9)$$

Proposed Excitation Model I

Tangent force:

- $f_{tan} = \theta(k) f_{max} \phi(t)$ (derived from [Bilbao, 2010])

- $\phi(t)$: raised cosine (1-4 ms)
- f_{max}: maximum force (10 N)
- $\theta(k)$: localization

Contact length [smp]:

- $e = l_c/h$

l_c: tangent width
h: grid spacing

Localization

if: $0 < e \leq 2$
Concentrated contact w/ linear interpolation

$$\theta(k) = \begin{cases} \frac{1}{e}, & k = k_i \\ \frac{b}{e}, & k = k_{i+1} \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Distributed contact w/ Lagrange interpolation (f-resp: -6 dB at Nyquist)

$$\theta(k) = \begin{cases} \frac{(e-1)(e-2)}{2}, & k = k_{i-1} \\ -e(e-2), & k = k_i \\ \frac{e(e-1)}{2}, & k = k_{i+1} \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

Proposed Excitation Model II

Stud force

- $f_{stud} = K(y_k)^\alpha \theta(x)$

- K: stiffness coefficient
- α : penetration coefficient

Resume

- Tangent contact: FDTD
- Wave propagation: DWG
- Secondary effects (e.g. beating, pickups): additional filters cascaded to the DWG

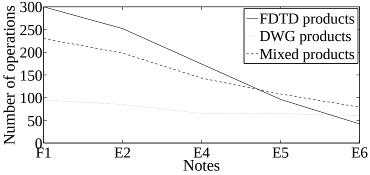
FDTD $\leftrightarrow$ DWG $\longrightarrow$ Secondary EFX

Simulations

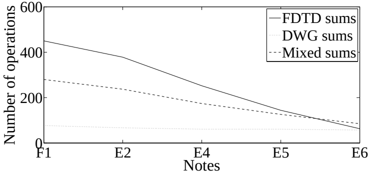
Video: Computer Simulation

Computational Cost

Products



Sums



Notes:

- DWG has a constant overhead due to constant filters order
- 50% string in FDTD and 50% in DWG

samples at http://a3lab.dii.univpm.it/projects/fdtd-adaptor

Conclusions

Outcome

- A novel SOFS-DWG adaptor is devised to extend previous FOFS-DWG modeling
- Reduced Computational Cost compared to FDTD
- Increased flexibility: excitation-related parameters are physical...
- ...but missing perceptual components can be added with standard DWG methods

Future Work

- Listening Tests similarly to [Gabrielli, 2011]
- Evaluate the mixed approach ease of parametrization

Thank you

References

References I

[1] M. Karjalainen, *1-d digital waveguide modeling for improved sound synthesis*, in Proc. Int. Conf. on Acoustics, Speech and Signal Processing (IEEE), vol. 2, pp. 1869-1872, Orland, USA, 2002.

[2] S. Bilbao and M. Rath, *Time domain emulation of the Clavinet*, in Proc. AES 128th Convention, London, UK, May 2010.

[3] C. Erkut, M. Karjalainen, *Finite Difference Method vs. Digital Waveguide Method in String Instrument Modeling and Synthesis*, report, online at https://www.acoustics.hut.fi/m̃ak/PUB/ISMA2002_Erkut.pdf

[4] L. Gabrielli, S. Squartini, and V. Välimäki, *A subjective validation method for musical instrument emulation*, in AES 131st Convention, NY, USA, 2011